Confidence limit for Population Proportion

① DEC 2011 / JAN 2012

A random sample of 700 units from a large consignment showed 200 were damaged. Find (i) 95% (ii) 99% confidence limits for the proportion of damaged units in the consignment.

Sol: Given

$n = 700$

no. of damaged units $= 200$

proportion of damaged units $p = \frac{200}{700}$

$p = \frac{200}{700} = 0.286$

$q = 1 - p = 1 - 0.286 = 0.714$

Standard error of proportion $\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.286)(0.714)}{700}}$

$\sigma_p = 0.017$ $p = 0.286$

(a) $C.L = 95\% \Rightarrow Z = 1.96$ $\boxed{p \pm Z\sigma_p}$

$p \pm Z\sigma_p \Rightarrow 0.286 \pm (1.96)(0.017)$

0.286 ± 0.033

$\Rightarrow [0.286 - 0.033 \;—\; 0.286 + 0.033]$

$\Rightarrow [0.253 \;—\; 0.319] \Rightarrow [25.3\% \;—\; 31.9\%]$

(b) $C.L = 99\% \Rightarrow Z = 2.58$

$p \pm Z\sigma_p \Rightarrow 0.286 \pm (2.58)(0.017)$

0.286 ± 0.044

$\Rightarrow [0.286 - 0.044 \;—\; 0.286 + 0.044]$

$\Rightarrow [0.242 \;—\; 0.330] \Rightarrow [24.2\% \;—\; 33\%]$

② In a random sample of 400 items from a large consignment 20 items were found to be defective. Find 99% confidence limits for the percentage of defectives in the consignment.

Sol:

$n = 400$

no. of defectives $= 20$

proportion of defectives

$p = \frac{20}{400} = 0.05$

$q = 1 - 0.05 = 0.95$

$C.L = 99\% \Rightarrow Z = 2.58$

$\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.05)(0.95)}{400}} = 0.011$

Confidence limits for proportion

$\boxed{p \pm Z\sigma_p}$

2021/3/31 12:18

$p \pm Z\sigma_p = 0.05 \pm (2.58)(0.011)$

$= 0.05 \pm 0.0284$

$\Rightarrow [0.05 - 0.0284 — 0.05 + 0.0284]$

$\Rightarrow [0.0216 — 0.0784] \Rightarrow [21.6\% — 78.4\%]$

③ In a sample of 400 oranges from a large consignment 40 were considered to be bad. Estimate the percentage of defective oranges in the whole consignment and assign limits which with in 95% confidence level.

Sol:

Sample = $n = 400$

no. of defectives $= 40$

proportion of defectives

$p = \frac{40}{400} = 0.1$

$q = 1 - p = 1 - 0.1 = 0.9$

C.L = 95%

$Z = 1.96$

$\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.1)(0.9)}{400}} = 0.015$

$\boxed{p \pm Z\sigma_p}$

$0.1 \pm (1.96)(0.015)$

0.1 ± 0.0294

$\Rightarrow 0.1 - 0.0294 — 0.1 + 0.0294$

$\Rightarrow 0.0706 — 0.1294$

$[7.06\% — 12.94\%]$

DEC 2010

④ Dr. Benjamin Shockey a noted social psychologist surveyed 150 top executives and found that 42% of them were unable to add fractions correctly.

ⓐ estimate the standard error of proportion

ⓑ Construct 99% confidence interval for the true population proportion of top executives who cannot add fractions correctly. ~~at~~ at least take three digit

Sol:

no. of executives $= n = 150$

prop of executives who cannot add $p = 42\%$

$p = 0.42$

$q = 1 - p = 0.42 = 0.58$

C.L = 99%

$Z = 2.58$

ⓐ $\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.42)(0.58)}{150}}$

$= \sqrt{0.001624}$

$\sigma_p = 0.0403$

2021/3/31

(b) $\boxed{p \pm z\sigma_p}$

$0.42 \pm (2.58)(0.0403)$

$\Rightarrow 0.42 \pm 0.10397$

$\Rightarrow 0.42 - 0.10397$ —— $0.42 + 0.10397$

$\Rightarrow [0.31603$ -- $0.52397]$ $\Rightarrow [31.6\%$ – $52.4\%]$

(5) When a sample of 70 retail executives was surveyed regarding the poor November performance of retail industry, 66 percent believed that decreased sales were due to unseasonably warm temperatures, resulting in consumers delaying purchase of cold weather items.

(a) Estimate the standard error of the proportion of retail executives who blame warm weather for low sales.

(b) Find upper and lower confidence limits for this proportion given at 95 percent confidence level.

Sol:

Given $n = 70$
$p = 66\% = 0.66$
$q = 1 - 0.66 = 0.34$
$C.L = 95$
$z = 1.96$

(a) $\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.66)(0.34)}{70}} = 0.0566$

(b) $\boxed{p \pm z\sigma_p}$

$0.66 \pm (1.96)(0.0566)$

0.66 ± 0.111

$\Rightarrow (0.66 - 0.111$ —— $0.66 + 0.111)$

$\Rightarrow [0.549$ – $0.771]$

$\Rightarrow$ lower limit 54.9%. upper limit 77.1%.

(6) April 2003

In a sample of 400 items, the number of defectives is found to be 5. Obtain 95% confidence interval for the population proportion of defectives.

Sol:

$n = 400$
no. of defectives $= 5$
$p = 5/400 = 0.0125$

$q = 1 - p = \cancel{0} - 0.0125$

$q = 0. \; 0.9875$

$\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.0125)(0.9875)}{400}} = 0.005555$

C.L $= 95\%$ $\quad z \Rightarrow 1.96$

$p \pm z\sigma_p$

$0.0125 \pm (1.96)(0.005555)$

0.0125 ± 0.0109

$[0.0016 - 0.0234] \Rightarrow [1.6\% - 2.34\%]$

(7) $\boxed{\text{NOV 2005}}$

out of consignment of 1,00,000 tennis balls, 400 were sorted out at random and examined and it was found that 20 of these were defective. How many defective balls can you reasonably expect to have consignment at 95% confidence level.

Sol:

1,00,000 = ?

$n = 400$

no of defectives = 20

$p = \frac{20}{400} = 0.05$

$q = 1 - p = 1 - 0.05 = 0.95$

$N = 1,00,000$

C.L $= 95\%$

$z = 1.96$

$\sigma_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.05)(0.95)}{400}} = 0.01089$

$\boxed{p \pm z\sigma_p}$

$0.05 \pm (1.96)(0.01089)$

0.05 ± 0.02134

$[0.02866 - 0.07134]$

Given consignment consists 1,00,000 balls

∴ ~~95% Confidence limits will be~~

$1,00,000 \times 0.02866 = 2866$

$1,00,000 \times 0.07134 = 7134$

with 95% we can expect the consignment will have in between [2866 — 7134] defective balls.

(NOV 2003)

5. Construct 90 percent Confidence interval for the estimate of the population mean on the basis of the following sample results.

$n = 16 \quad \bar{x} = 98 \quad s = 1.5 \quad$ and $\quad t(15, 0.10) = 1.753$

Sol: Given $n = 16$

$\bar{x} = 98$

$s = 1.5$

Conf. level = 90%.

$df = n-1 = 16-1 = 15$

$t(15, 0.10) = 1.75$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ or $\frac{s}{\sqrt{n}} = \frac{1.5}{\sqrt{16}}$

$= \frac{1.5}{4} = 0.375$

Confidence limits $\boxed{\bar{x} \pm t\,\sigma_{\bar{x}}}$

$98 \pm (1.753)(0.375)$

98 ± 0.657

$[97.343 — 98.657]$

(MAY 2008)

6. The sample mean computed from a random sample (r.s) of size 50 drawn from a population has the value 52.5. Suppose that the population standard deviation is known to be equal to 16. Find (a) an estimate standard error of mean (b) a 95% C.I of the pop mean

Sol: Sample size $n = 50$

Sample mean $\bar{x} = 52.5$

Pop S.D $\sigma = 16$

Conf. level $= 95\%$

(a) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{50}} = \frac{16}{7.071} = 2.263$

(b) Conf. limits $\boxed{\bar{x} \pm Z\,\sigma_{\bar{x}}}$

$52.5 \pm (1.96)(2.263)$

52.5 ± 4.4354

$[48.0646 — 56.4354]$

Confidence interval $\bar{x} \pm t\sigma_{\bar{x}}$

$62 \pm (2.201)(2.8867)$

62 ± 6.354

$62 - 6.354$ —— $62 + 6.354$

$[55.646 \; -- \; 68.354]$

3. The following sample of 8 observations is from an infinite population with a normal distribution.

أي عنصر بيتربع في عدد

75.3 76.4 83.2 91.0 80.1 77.5 84.8 81.0

(a) find sample mean. (b) Estimate the population S.D
(c) construct 98% confidence interval for the pop mean

Sol: $n = 8$ $\Sigma x = 649.3$

$\bar{x} = \frac{\Sigma x}{n} = \frac{649.3}{8} = 81.1625$ $\Sigma x^2 =$

$$\boxed{s^2 = \frac{\Sigma x^2 - n(\bar{x})^2}{n-1}} = \frac{52884.597 - 8(81.1625)^2}{8-1}$$

$$= \frac{52884.597 - 8(6587.3514)}{7}$$

$$\cancel{\sigma = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}} \qquad = \frac{52884.597 - 52698.81125}{7}$$

$\boxed{\text{degrees of freedom } df = n-1}$ $s^2 = \frac{185.7788}{7} = 26.5398$

$s = \sqrt{26.5398} = 5.15$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{5.15}{\sqrt{8}} = \frac{5.15}{2.8284} = 1.820$

$df = n-1 = 8-1 = 7$

$C.L = 98\%$ $t = 2.998$

$\bar{x} \pm t\sigma_{\bar{x}} = 81.1625 \pm (2.998)(1.82)$

81.1625 ± 5.4564

$[75.7061 \; —— \; 86.6189]$

2021/3/31 12:1

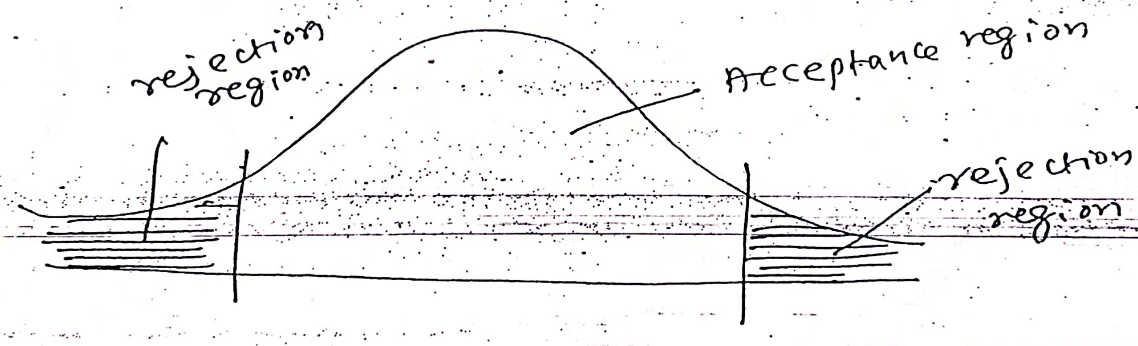
Hypothesis: ~~Anass~~ assumption about a population parameter is called a Hypothesis

Null Hypothesis (H_0) An assumption about a population parameter we wish to test is called Null Hypothesis

Alternative Hypothesis (H_1): The conclusion we make about a population parameter, when the data fails to support Null hypothesis is called Alternative Hypothesis

Two Tailed Test of Hypothesis:

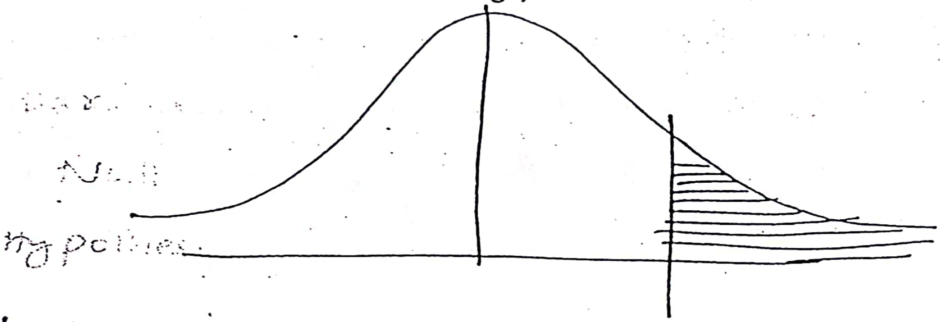
A Test of hypothesis in which we reject a null hypothesis when sample value is significantly very higher or significantly very lower than the hypothesized value of the population parameter is called Two Tailed Test of hypothesis



(OR) A Test of hypothesis in which there are two rejection regions is called Two Tailed Test of hypothesis

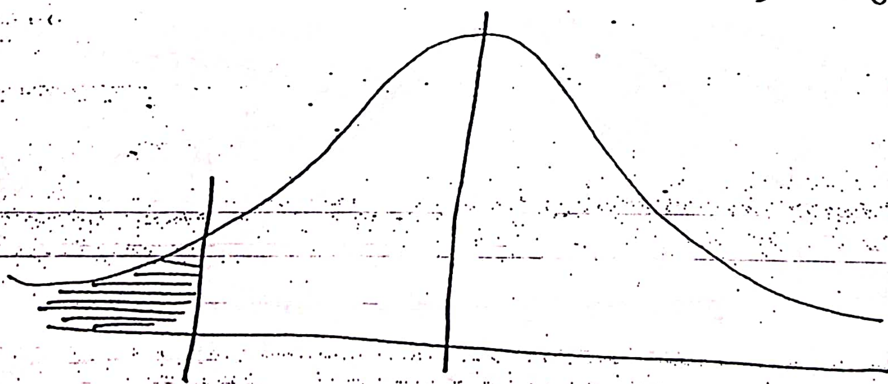
Right Tailed Test or Upper Tailed Test of hypothesis:

A Test of hypothesis in which we reject a null hypothesis when the sample value is significantly very higher than the hypothesized value of the population parameter is called Right tailed test of hypothesis.



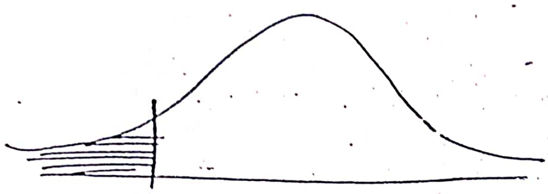
Left Tailed Test of Hypothesis: (Lower Tailed Test)

A test of hypothesis in which we reject a null hypothesis when the sample value is significantly very lower than the hypothesized value of the population parameter is called Left tailed test of hypothesis

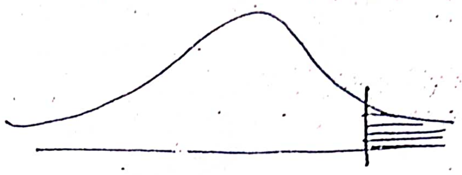


One Tailed Test of Hypothesis

A Test of hypothesis in which there is only one rejection region is called one Tailed Test of hypothesis.



Left Tailed test



Right Tailed Test.

(*) **Significance level (α)**: A value indicating the percentage of sample values that lie outside certain limits, beyond which we reject a null hypothesis is called significance level.

(OR) Probability of rejecting a null hypothesis when it is true is called significance level.

Critical value: The value of the standard statistic beyond which we reject the null hypothesis or the boundary between the acceptance region and rejection region is called critical value.

(*) **Type I error (α)**: Reject a null hypothesis when it is True is called Type I error.

Type II error : (β)

Accepting the null hypothesis when it is False is called Type II error.

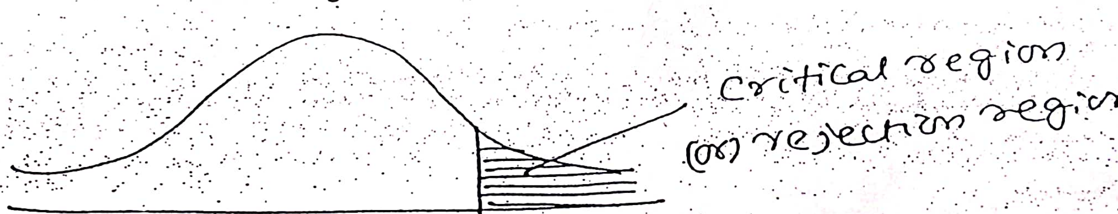
Power of the hypothesis Test:

The probability of rejecting the null hypothesis when it is False, That is a measure of how well the hypothesis test is working.

It is calculated as $1-\beta$

(*) Critical region:

A region (Corresponding to a statistic) in the sample space which amounts to rejection of Ho is termed as rejection or critical region.



CRITICAL VALUES

Test \ α	10%	5%	1%
Two Tail	$\lvert Z_\alpha \rvert = 1.645$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 2.58$
Right Tail	$Z_\alpha = 1.28$	$Z_\alpha = 1.645$	$Z_\alpha = 2.33$
Left Tail	$Z_\alpha = -1.28$	$Z_\alpha = -1.645$	$Z_\alpha = -2.33$

UNIT – III

(PROBLEMS)

* TEST NO: 1

1ML

when it is mean then its Z

Test of significance of Mean

(one sample) (Large sample) ($n \geq 30$) ('Z' test)

① The mean Life of a sample of 100 electric bulbs produced by a Company is found to be 1570 hrs. with a S.D of 120 hrs. If μ is the mean Life time of all the bulbs produced by the Company, test the hypothesis that $\mu = 1600$ against the Alternative hypothesis ($\mu \neq 1600$) using Alfa $\alpha = 0.05$

Sol:

Given $n = 100$

$\bar{x} = 1570$

$s = 120$

$\mu = 1600$

I

H_0 : $\mu = 1600$

H_1 : $\mu \neq 1600$ (T.T 'Z' test)

II Computation of Test statistic (C.T.S) — Yes!

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$s/\sqrt{n} = \frac{120}{\sqrt{100}} = \frac{120}{10} = 12$$

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1570 - 1600}{12} = \frac{-30}{12} = -2.5$$

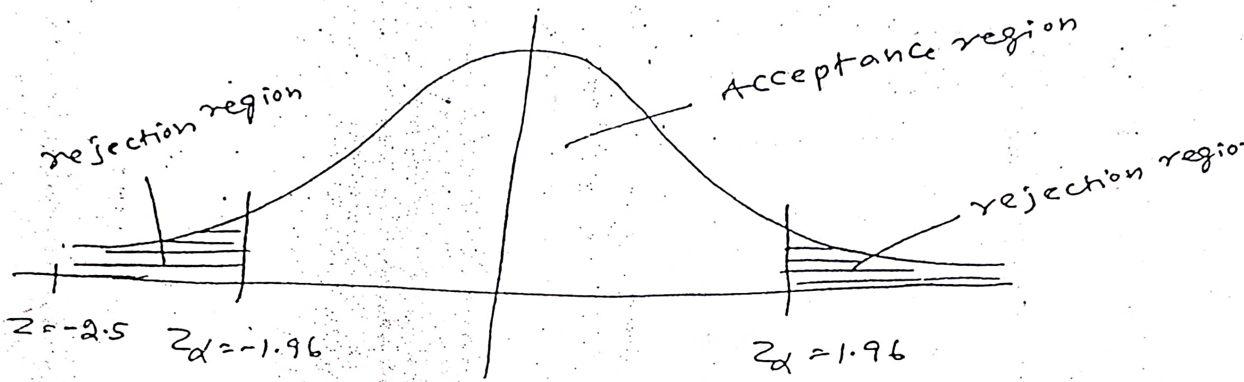
III L.O.S

$\alpha = 0.05$

IV C.V

Critical value for TT z-test at $\alpha = 0.05$ is

$|Z_\alpha| = 1.96$



(V) Decision

Since Z lies in rejection region Null hypothesis is rejected and Alternative hypothesis is accepted with 5% L.o.s

$\therefore \mu \neq 1600.$

(2) An Automatic machine was designed to pack exactly 2.0 kg of vanaspati. A sample of 100 tins was examined to test the machine. The average weight was found to be 1.94 Kg with S.D 0.10 Kg. Is the machine working properly

(23)

Sol:

Given: $\mu = 2.0$ Kg

$n = 100$

$\bar{x} = 1.94$ Kg.

$S = 0.10$ Kg.

I

H_0: The machine is working properly.
$\mu = 2.0$

H_1: The machine is not working properly.
$\mu \neq 2.0$ (TT 'z' test)

II C.T.S

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad \text{or} \quad Z = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

$$S/\sqrt{n} = \frac{0.10}{\sqrt{100}} = \frac{0.10}{10} = 0.010$$

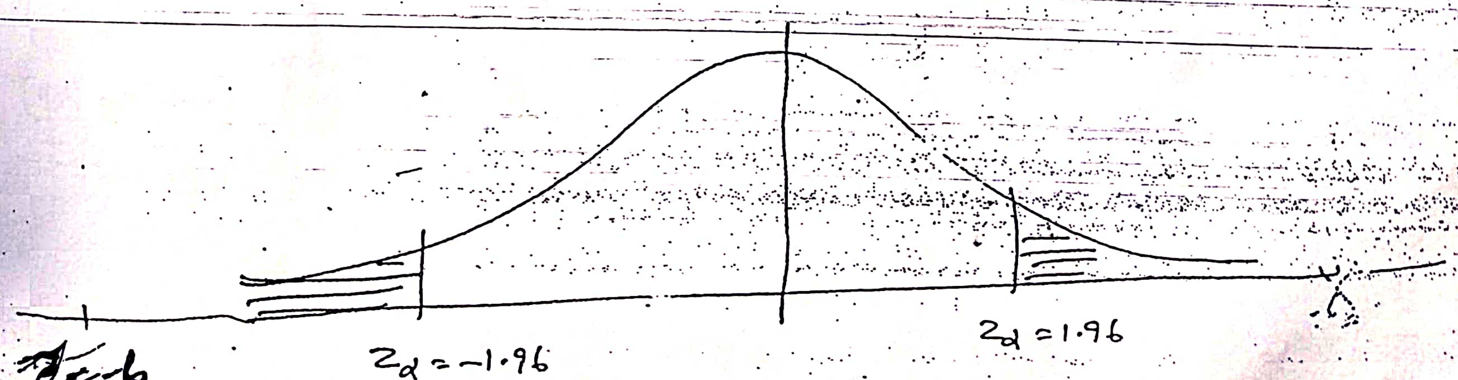
$$Z = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{1.94 - 2.0}{0.010} = \frac{-0.06}{0.01} = -6$$

III L.O.S

$\alpha = 5\% = 0.05$

IV C.V

C.V for T.T 'z' test at 5% $|Z_\alpha| = 1.96$



V Decision:

Since z lies in rejection rejection Ho is rejected and H_1 is accepted with 5% L.O.S

∴ The machine is not working properly.

3. A sample of 900 members has a mean of 3.4 cms and S.D 2.61 cms. Is the sample from a population with mean 3.25 cm at $\alpha = 0.05$?

Sol:

Given $n = 900$

$\bar{x} = 3.4$

$s = 2.61$

$\mu = 3.25$

$\alpha = 0.05$

I

H_0: There is no significance difference between sample mean and Population mean $\mu = 3.25$ [Sample has come from the population]

H_1: There is a significance difference between sample mean and pop mean $\mu \neq 3.25$ (TT Z test) [Sample has not come from the pop]

II C.T.S

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

2021/3/31 12:21

$$S/\sqrt{n} = \frac{2.61}{\sqrt{900}} = \frac{2.61}{30} = 0.087$$

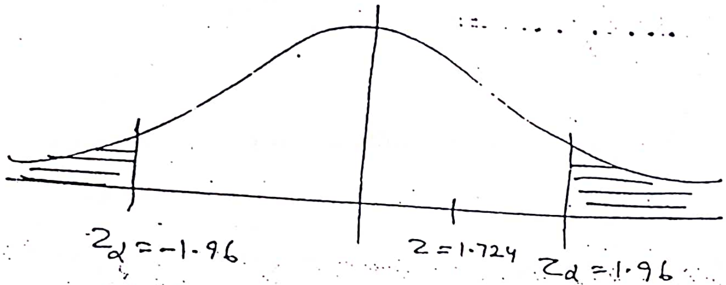
$$Z = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{3.4 - 3.25}{0.087} = \frac{0.15}{0.087} = 1.724$$

III <u>L.O.S</u>

$\alpha = 0.05$

IV <u>C.V</u>

C.V for TT 'z' test at $\alpha = 0.05$ is $|Z_\alpha| = 1.96$



V <u>Decision</u>

Since 'z' value lies in acceptance region Null hypothesis is accepted.

∴ The sample is from the pop having mean 3.25

HW

(4) # A random sample of 100 students gave a mean ~~of~~ weight of 58 kgs with a S.D of 4 kg. Test the Hypothesis that the mean weight in the population is 60 kg

<u>Hint</u>:

$n = 100$ $\quad H_0: \mu = 60$

$\bar{x} = 58$ $\quad H_1: \mu \neq 60$ (TT 'z' test)

$s = 4$ kg

$\mu = 60$ $\quad z = -5$

$\alpha = 0.05$ $\quad \boxed{\text{REJECT}}$

HW 5

It is claimed that a random sample of 100 tyres with a mean life of 15269 Km is drawn from a population of tyres which has a mean life of 15200 Kms and S.D of 1248 Kms. Test the validity of the claim?

Hint:

$n = 100$ $\quad$ $H_0: \mu = 15200$

$\bar{x} = 15269$ $\quad$ $H_1: \mu \neq 15200$ (TT 'z' test)

$\mu = 15200$

$\sigma = 1248$ $\quad$ $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = 0.553$

$\boxed{\text{ACCEPTED}}$

6 (2011)

The mean breaking strength of the cables supplied by a manufacturer is 1800 with S.D of 100. By a new technique on the manufacturing process, it is claimed that the breaking strength of the cables have increased. In order to test this claim, a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% L.O.S

Sol:

Given $\mu = 1800$

$\sigma = 100$

$n = 50$

$\bar{x} = 1850$

$\alpha = 0.01$

I. Ho: There is no significance difference in mean breaking strength of the cables after using new technique in the manufacturing process

$\mu = 1800$

H_1: The mean breaking strength of the cables has increased after using new technique in the manufacturing process

$\mu > 1800$ (RT 'z' test)

II C.T.S

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \sigma/\sqrt{n} = \frac{100}{\sqrt{50}} = \frac{100}{7.07} = 14.144$$

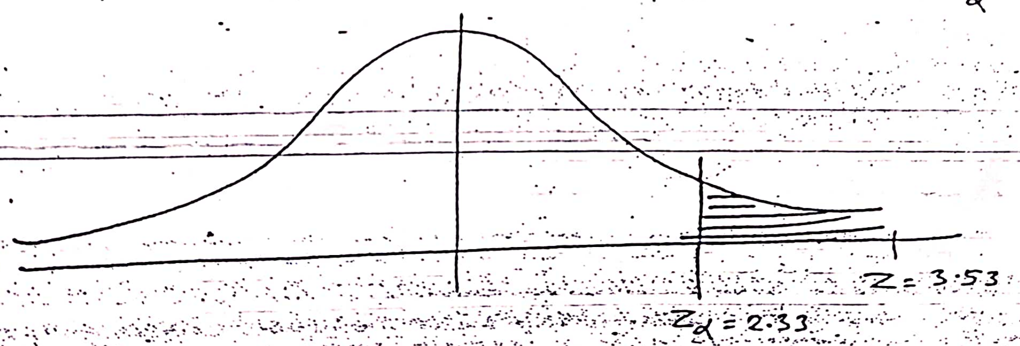
$$Z = \frac{1850-1800}{14.144} = \frac{50}{14.144} = 3.53$$

III L.O.S

$\alpha = 1\% = 0.01$

IV C.V

C.V for RT 'z' test at $\alpha = 0.01$ $Z_\alpha = 2.33$



V Decision : Since z lies in rejection region Null hypothesis is rejected and Alternative hypothesis is accepted with $\alpha = 0.01$

$\therefore$ The mean breaking strength of the cables has increased after using the new technique.

* (7) American Theatres knows that a certain hit movie ran an average of 84 days in each city, and the corresponding standard deviation was 10 days. The manager of the South eastern district was interested in comparing movie's popularity in his region with that in all of American's other Theatres. He randomly chose 75 theatres in his region and found that they ran the movie an average of 81.5 days.

(a) State appropriate hypothesis for testing whether there was a significant difference in the length of the picture's run between theatres in the south eastern district and all of Americans other theatres.

(b) At a 1% L.o.s test these hypothesis

Given: $\mu = 84$ days

$\sigma = 10$

$n = 75$

$\bar{x} = 81.5$ days

$\alpha = 0.01$

a) H_0: There is no significance difference in the length of the pictures run between theatres in South eastern district and all of Americans other theatres

H_1: There ~~a~~ is a significance difference in the length of the picture's run between theatres in South eastern district and all of American's other theatres

HINT:

b) I

$H_0: \mu = 84$

$H_1: \mu \neq 84$ (TT 'z' test)

II

(ACCEPT)

$Z = -2.17$; $Z_\alpha = \pm 2.58$

EXAM

8. (*) (*)

Hinton Press hypothesizes that the average life of its largest web press is 14,500 hrs. They know that the standard deviation of press life is 2,100 hrs. From a sample of 25 presses, the Company finds ~~that~~ a sample mean of 13,000 hrs. At a 0.01 L.O.S, should the Company conclude that the average life of the presses is Less than the hypothesized 14,500 hrs.

Sol: Given $\mu = 14500$

$\sigma = 2100$

$n = 25$

$\bar{x} = 13,000$

$\alpha = 0.01$

I. H_0: $\mu = 14,500$

The average life of presses is 14,500

H_1: $\mu < 14,500$ (LT 'z' test).

The average life of presses is Less than 14,500 hrs

II. <u>C.T.S</u>

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \sigma/\sqrt{n} = \frac{2100}{\sqrt{25}} = \frac{2100}{5} = 420$$

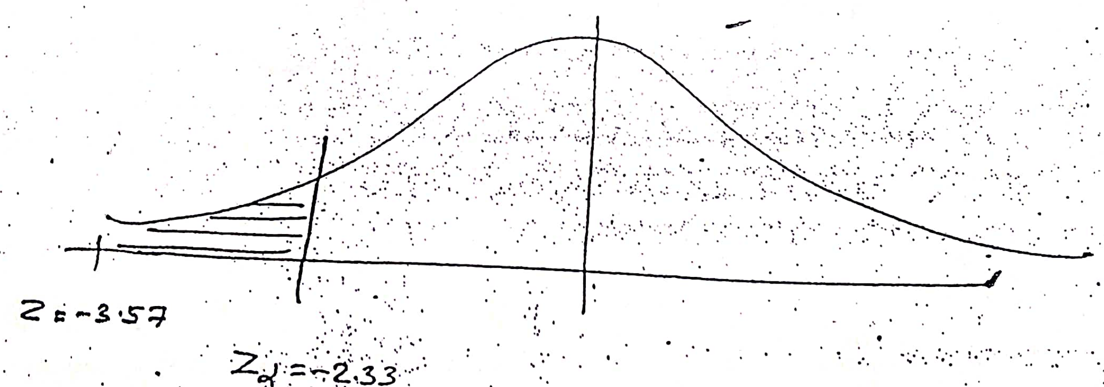
$$Z = \frac{13000 - 14500}{420} = \frac{-1500}{420} = -3.57$$

III. <u>L.O.S</u>

$\alpha = 0.01$

IV. <u>C.V</u>

C.V for LT 'z' test at $\alpha = 0.01$: $Z_\alpha = -2.33$



V. <u>Decision</u>: when it is not accept

Since Z lies in rejection region H_0 is rejected and H_1 is accepted with $\alpha = 0.01$

∴ The average life of presses is less than 14,500 hrs.

NOV 2004 | MAY 2011

(9) A sample of 450 items is taken from a population whose standard deviation is 20. The mean of the sample is 30. Test whether the sample has come from the population with mean 29 at 5% level of significance

Sol: Given sample size $n = 450$

pop S.D $\sigma = 20$

sample mean $\bar{x} = 30$

pop mean $\mu = 29$

L.O.S $\alpha = 0.05$

I

Ho: The sample has come from the population having mean 29

$\mu = 29$

H_1: The sample has not come from the pop having mean 29

$\mu \neq 29$ (CTT z test)

II. C.T.S

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \qquad \sigma/\sqrt{n} = \frac{20}{\sqrt{450}} = \frac{20}{21.2132} = 0.943$$

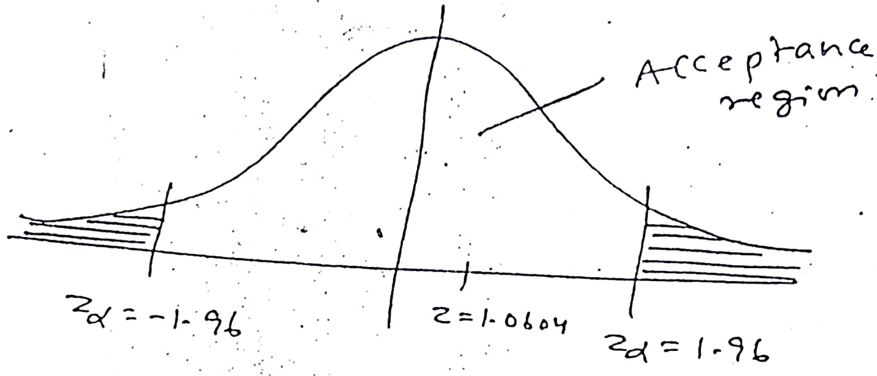
$$Z = \frac{30 - 29}{0.943} = \frac{1}{0.943} = 1.0604$$

III L.O.S

$\alpha = 0.05$

IV C.V

C.V for TT 'z' test at $\alpha = 0.05$ is $|Z_\alpha| = 1.96$



V Decision: when it is accept

Since 'z' lies in Acceptance region Ho is accepted with 5% L.O.S

∴ The sample has come from the pop with mean 29.

Test of Significance of Proportion

(One Sample) (Large Sample) ($n \geq 30$)

[Z test]

DEC/JAN 2012

(1) In a Sample of 400 parts manufactured by a factory the number of defective parts was found to be 30. The company however claimed that only (5%) of their product is defective. Is the claim tenable?

$P = 5\%$

Sol:

$n = 400$

no of defective parts = 30

proportion of defective parts $= \overset{\text{small } p}{p} = \frac{30}{400} = 0.075$

$p = 0.075$

capital P

$\boxed{P = 5\% = 0.05}$ $\quad Q = 1 - P = 1 - 0.05 = 0.95$

I   Null hypothesis $H_0: P = 0.05$

Alternative Hypothesis $H_1: P > 0.05$ (RT. 'z' test)

II. Computation of Test statistic

$$Z = \frac{p - P}{\sqrt{PQ/n}}$$

$$Z = \frac{(0.075) - (0.05)}{\sqrt{\frac{(0.05)(0.95)}{400}}} \qquad \frac{0.025}{0.011}$$

2021/3/31 12:22

$$Z = \frac{0.075 - 0.05}{\sqrt{0.000187}}$$

$$Z = \frac{0.025}{0.011}$$

$$Z = 2.27$$

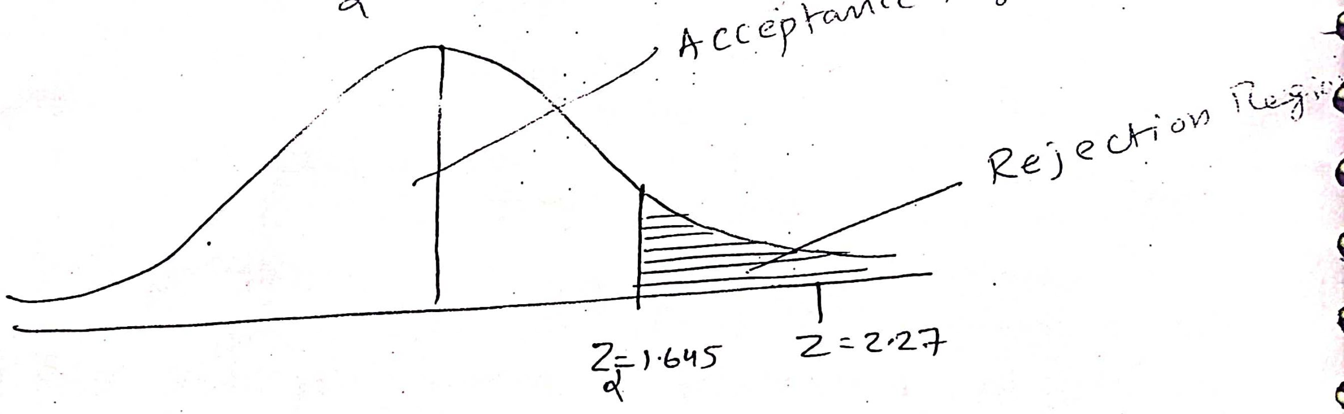
III. Level of Significance (L.O.S)

$\alpha = 5\%$ $\alpha = 0.05$

IV Critical value

Critical value for RT 'Z' test 5% LOS

$Z_{\alpha} = 1.645$



V. Decision:

Since Z lies in Rejection Region

Null Hypothesis is Rejected and

Alternative hypothesis is accepted.

(2) A wholesaler in apples claims that only 4% of the apples supplied by him are defective. A random sample of 600 apples contained 36 defective apples. Test the claim of the whole saler?

Sol: 4%

$n = 600$

defective $= 36$

$p = \frac{36}{600} = 0.06$

$P = 4\% = 0.04$

$Q = 1 - 0.04 = 0.96$

I $H_0 : P = 0.04$

$H_1 : P_1 > 0.04$ (RT 'Z' test)

II C.T.S

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.06 - 0.04}{\sqrt{\frac{(0.04)(0.96)}{400}}}$$

$$= \frac{0.02}{\sqrt{0.000096}}$$

$$= \frac{0.02}{0.009798}$$

$$= 2.04$$

2021/3/31 12:23

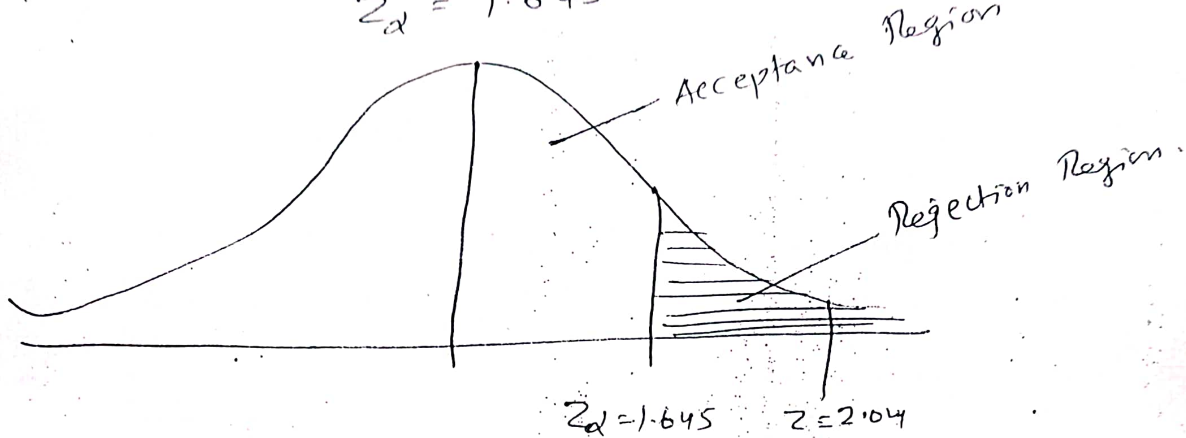
III <u>L.O.S</u> $\alpha = 5\%$

$\alpha = 0.05$

IV <u>C.V</u>

C.V for RT 'z' test at $\alpha = 5\%$.

$Z_\alpha = 1.645$



V <u>Decision</u>:

Since Z lies in Rejection Region

H_0 rejected & H_1 accepted.

3. In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

Sol: Given

325 men out of 600 are smokers.

$n = 600,$

Smokers $= 325$

$p = \frac{325}{600} = 0.5417$

$$\boxed{P = 0.5 \quad Q = 1-P \quad Q = 1-0.5 = 0.5}$$?? [Majority]

I. $H_0: P = 0.5$ [Smokers & Non Smokers are equal in city]

$H_1: P > 0.5$ (RT Z test)

II. C.T.S

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.5417 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{600}}}$$

$$= \frac{0.0417}{\sqrt{0.0004165}}$$

$$= \frac{0.0417}{0.0204}$$

$$Z = 2.04$$

2021/3/31 12:23

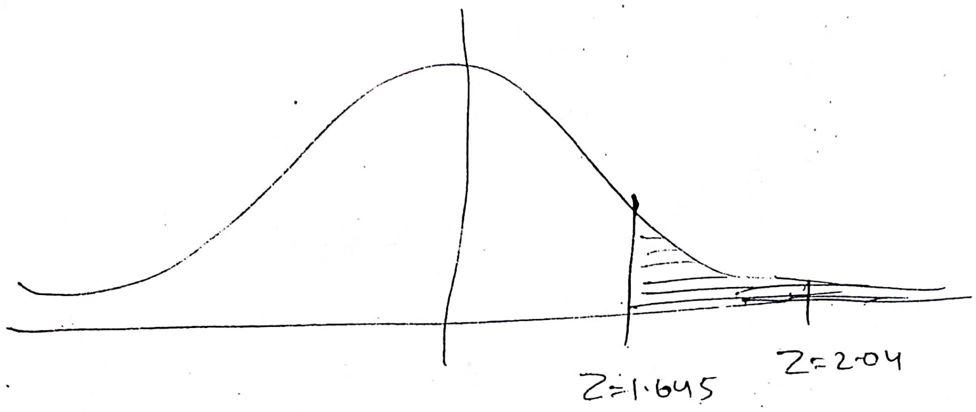
III L.O.S

$\alpha = 5\%$

IV C.V

Critical value for RT Z test at 5%

$Z = 1.645$



V Decision

Since Z lies in rejection region

H_0 rejected and H_1 accepted.

∴ Majority of Men are Smokers in the city.

(5) A ketchup manufacturer is in the process of deciding whether to produce a new extra spicy brand. The company's marketing research department used a national ~~survey~~ telephone survey of 6000 households and found that the extra spicy brand would be purchased by 335 of them. A much more study made 2 years ago showed that 5 percent of the household would purchase the brand then. At a 1% L.O.S should the company conclude that there is an increased interest in the extra spicy brand?

Sol. $n = 6000$

interested to purchase $= 335$

$$p = \frac{335}{6000} = 0.05583$$

$P = 5\% = 0.05$

$Q = 1 - 0.05 = 0.95$

I $H_0 : P = 0.05$

$H_1 : P > 0.05$ (RT z test)

II C.T.S

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.05583 - 0.05}{\sqrt{\frac{(0.05)(0.95)}{6000}}}$$

$$= \frac{0.00583}{0.002813}$$

$$z = 2.0725$$

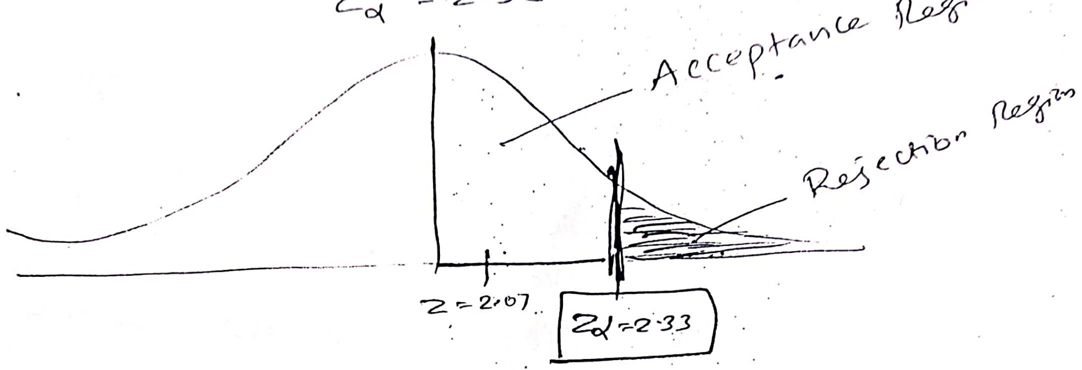
III **L.O.S**

$\alpha = 1\%$ or $\alpha = 0.01$

IV **C.V**

C.V for RT 'Z' test at 1% L.O.S

$Z_\alpha = 2.33$



V **Decision:**

Since Z lies in Acceptance region

$\underline{H_0}$ is <u>accepted</u>.

∴ There is no increased interest in the extra spicy brand.

NOV 2009 (*) (*) (*) (*) (*)

(1) In a random Sample of 400 persons from a Large population 120 are females can it be said that males and females are in the ratio of 5:3 in the Populations? Use 1% Level of signifinace?

Sol.

Sample size $n = 400$

no of females $= 120$

proportion of females $= p = \frac{120}{400} = 0.30$

we have to test males and females in the ratio 5:3

males : females

5 : 3

Total $5+3 = 8$

female ration $= \frac{3}{8}$

Proportion female ration $P = \frac{3}{8}$

$P = \frac{3}{8} = 0.375$

I

Null Hypothesis (H_0): Males and females in the population are in the ration 5:3

(or) $H_0: P = 0.375$

Alternative Hypothesis (H_1): Males and ~~females in the~~ Population are not in the ratio 5:3

or $H_1: P \neq 0.375$ (TT z test)

2021/3/31 12:23

II **C.T.S**

$$Z = \frac{p - P}{\sqrt{PQ/n}}$$

$P = 0.375$

$Q = 1 - P$

$= 1 - 0.375$

$Q = 0.625$

$$\sqrt{\frac{PQ}{n}} = \sqrt{\frac{(0.375)(0.625)}{400}} = 0.024$$

$$Z = \frac{p - P}{\sqrt{PQ/n}} = \frac{0.30 - 0.375}{0.024} = \frac{-0.075}{0.024} = -3.125$$

$$Z = -3.125$$

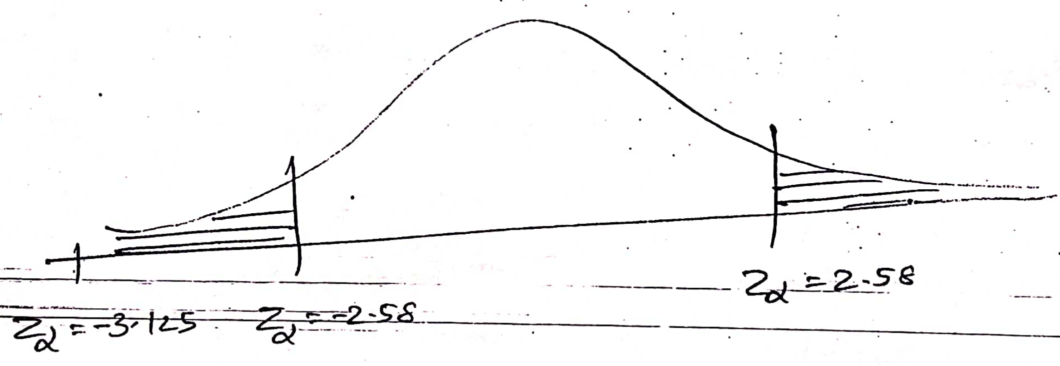
III **L.O.S**

$\alpha = 1\%$

IV **Critical value (C.V)**

C.V for Two Tail 'Z' test at 1% L.O.S

$$|Z_\alpha| = 2.58$$



V **Decision:** Since 'Z' lies in Rejection region

Ho. rejected and H_1 accepted.

∴ Males and females in the population are not in the ratio 5:3.

TEST NO: 3

Test of significance of mean

(one sample) (small sample) [$n < 30$]

$\boxed{\sigma}$ is not known [t test]

C.T.S

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$

degrees of freedom

$$\boxed{df = n - 1}$$

(1) Given a sample mean of 83, a sample S.D of 12.5 and a sample size of 22, test the hypothesis that the value of the population mean is 70 against the alternative hypothesis that it is more than 70. use 0.025 significance level.

Sol:

Given $\bar{x} = 83$

$s = 12.5$

$n = 22$

$H_0 : \mu = 70$

$H_1 : \mu > 70$

$\alpha = 0.025$

I. $H_0 : \mu = 70$

$H_1 : \mu > 70$ (RT 't' test)

II. C.T.S

$$s/\sqrt{n} = \frac{12.5}{\sqrt{22}} = \frac{12.5}{4.6904} = 2.665$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{83-70}{2.665} = \frac{13}{2.665} = 4.878$$

III L.O.S

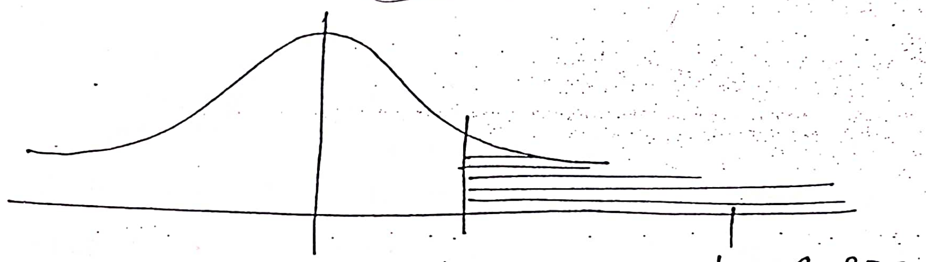
$\alpha = 0.025$

IV C.V

$df = n-1 = 22-1 = 21$

C.V for RT 't' test at 21 df at $\alpha = 0.025$

is $t_\alpha = 2.080$



V Decision

Since 't' lies in rejection region Ho is rejected and H_1 is accepted with $\alpha = 0.025$

∴ pop mean is more than 70.

(2) Given a sample mean of 94.3, a sample S.D of 8.4 and a sample size of 6, test the hypothesis that the value of the population mean is 100 against the alternative hypothesis that it is Less than 100. Use the 0.05 significance level

Sol : Given $\bar{x} = 94.3$

$s = 8.4$

Sample size $n = 6$

$H_0 : \mu = 100$

$H_1 : \mu < 100$

$\alpha = 0.05$

I

$H_0 : \mu = 100$

$H_1 : \mu < 100$ (LT 'z' test)

II C.T.S

$s/\sqrt{n} = \frac{8.4}{\sqrt{6}} = \frac{8.4}{2.4495} = 3.4293$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{94.3 - 100}{3.4293} = \frac{-5.7}{3.4293} = 1.6621$$

III L.O.S

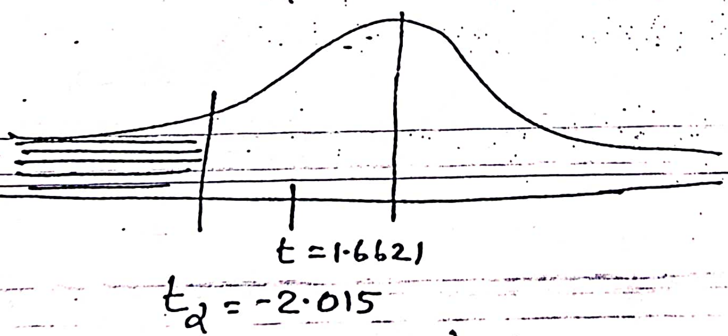
$\alpha = 0.05$

IV C.V

$df = n-1 = 6-1 = 5$

C.V for LT 't' test at 5 df at $\alpha = 0.05$

$$t_\alpha = -2.015$$



V **Decision** : Since 't' lies in acceptance region

Ho is accepted with 5% L.O.S

∴ Pop mean is equal to 100.

[NOV 2005]

(3) A machine is designed to produce insulating washers for electrical devices of average thickness of 0.25 cm. A random sample of 10 washers was found to have an average thickness of 0.24 cm with a S.D of 0.002 cm. Test the significance of the deviation [use 5% Level Table value 2.262]

Sol: Given $\mu = 0.25$ cm

$n = 10$

$\bar{x} = 0.24$ cm

$s = 0.002$ cm

$\alpha = 5\%$.

I

$H_0: \mu = 0.25$

$H_1: \mu \neq 0.25$ [TT 't' test]

II C.T.S

$$s/\sqrt{n} = \frac{0.002}{\sqrt{10}} = \frac{0.002}{3.1623}$$

$$= 0.000632$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t = \frac{0.24 - 0.25}{0.000632} = \frac{-0.01}{0.000632} = -15.8227$$

III L.O.S

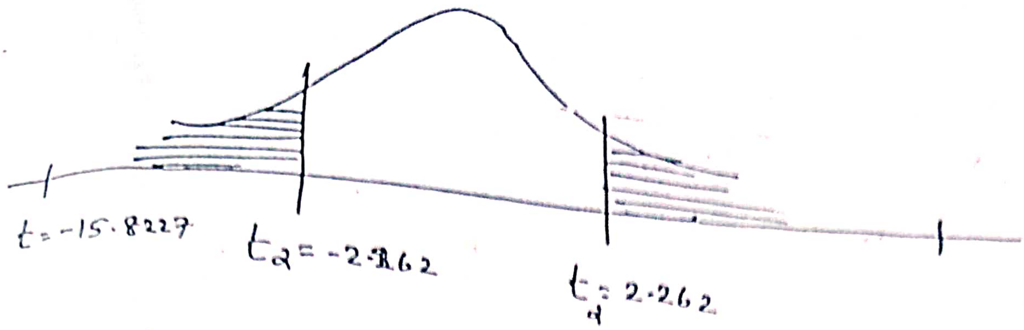
$\alpha = 0.05$

IV C.V

$df = n-1 = 10-1 = 9$

C.V for TT 't' test at 9 df at 0.05 is

$$|t_\alpha| = 2.262$$



V Decision : since t lies in rejection region Ho rejected and H_1 accepted with $\alpha = 0.05$

$\therefore \mu \neq 0.25$

NOV 2001

(4) A Sample of 10 circular discs, manufactured by a Company has an average diameter of 2.5 inches. Assuming diameter follows a normal distribution with variance 0.25 inches, ~~Ho~~ test whether the average diameter of the discs manufactured is 2.25 inches.

Sol :

$$n = 10$$
$$\bar{x} = 2.5$$
$$s^2 = 0.25$$
$$\mu = 2.25$$

I

$H_0 : \mu = 2.25$

$H_1 : \mu \neq 2.25$ (TT 't' test)

II C.T.S

$$s/\sqrt{n} = \sqrt{\frac{s^2}{n}} = \sqrt{\frac{0.25}{10}}$$
$$= 0.1581$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{2.5 - 2.25}{0.1581} = \frac{0.25}{0.1581} = 1.5813$

III <u>L.O.S</u>

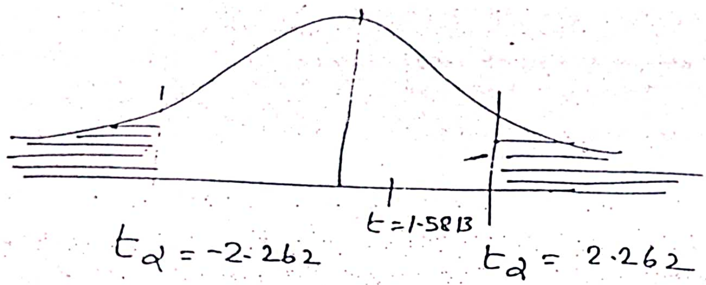
$\alpha = 0.05$

IV <u>C.V</u>

$df = n - 1 = 10 - 1 = 9$

C.V for TT 't' test at 9 d.f at $\alpha = 0.05$

$|t_\alpha| = 2.262$



V <u>Decision</u> Since 't' lies in acceptance region Ho is accepted with 5% L.O.S

∴ average diameter of the discs manufactured is 2.25.

(5) If a sample of 25 observations reveals a sample mean of 52 and a sample variance of 4.2 test the hypothesis that the population mean is 65 against the alternative hypothesis that is <u>some other value</u> use 0.01 significance level.

Hint :

$n = 25$

$\bar{x} = 52$

$s^2 = 4.2$

$H_0 : \mu = 65$

$H_1 : \mu \neq 65$

$\alpha = 0.01$

$df = n-1 = 25-1 = 24$

$|t_\alpha| = 2.797$

(6) Ten cartons are taken at random from an automatic filling machine. The mean net weight of the 10 cartons is 11.8 Kgs with a S.D of 0.15 Kgs. Does the sample mean differ significantly from the intended weight of 12 Kgs use $\alpha = 0.05$

Hint :

$n = 10$

$\bar{x} = 11.8$

$s = 0.15$

$\mu = 12$

$\alpha = 0.05$

$H_0 : \mu = 12$

$H_1 : \mu \neq 12$ (TT t test)

$df = n-1 = 10-1 = 9$

$|t_{0.05}| = 2.262$

(REJECT)

(7) The mean weekly sales of chocolate bar in candy stores is 146.3 bars per store. After an advertising campaign the mean sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2 Is the advertising campaign successful?

Hint :

$\mu = 146.3$

$n = 22$

$\bar{x} = 153.7$

$s = 17.2$

$H_0 : \mu = 146.3$

$H_1 : \mu > 146.3$ (RT t test)

(Adv is successful)

$df = 22-1 = 21$

(8) APRIL 2005

Ten individuals are choosen at random from a normal population and their heights are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71 inches. Test if the sample belongs to the population whose mean height is 66 inches (Table value 2.262)

Sol: Given $n = 10$

63, 63, 66, 67, 68, 69, 70, 70, 71, 71

$$\bar{x} = \frac{\Sigma x}{n} = \frac{678}{10}$$

$$\bar{x} = 67.8$$

x	x^2
63	3969
63	3969
66	4356
67	4489
68	4624
69	4721
70	4900
70	4900
71	5041
71	5041
$\Sigma x = 678$	$\Sigma x^2 = 46010$

$$\boxed{s^2 = \frac{\Sigma x^2 - n(\bar{x})^2}{n-1}}$$

$$s^2 = \frac{46010 - 10(67.8)^2}{10-1}$$

$$s^2 = \frac{46010 - 10(4596.84)}{9}$$

$$s^2 = \frac{46010 - 45968.4}{9}$$

$$s^2 = \frac{41.6}{9}$$

$$s^2 = 4.6222$$

$$s = \sqrt{s^2} = 2.1499$$

$\boxed{\mu = 66}$ Given

$\mu = 66$

I $H_0 : \mu = 66$

$H_1 : \mu \neq 66$ (TT 't' test)

II <u>C.T.S</u>

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \qquad s/\sqrt{n} = \frac{2.1499}{\sqrt{10}} = 0.6798$$

$$t = \frac{67.8 - 66}{0.6798} = \frac{1.8}{0.6798} = 2.6478$$

III <u>L.O.S</u>

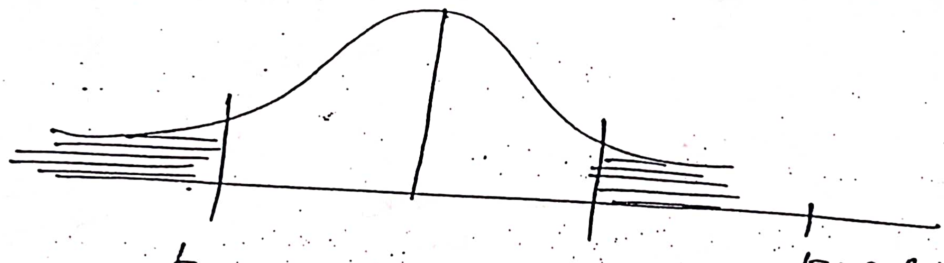
$\alpha = 0.05$

IV <u>C.V</u>

$df = n-1 = 10-1 = 9$

C.V for TT 't' test at 9 df at $\alpha = 0.05$

$(t_\alpha) = 2.262$



$t_\alpha = -2.262$ $\quad t_\alpha = 2.262$ $\quad t = 2.6478$

V <u>Decision</u>

Since t lies in rejection region Ho rejected and H_1 accepted with 5% L.O.S

∴ The sample does not belongs to the population whose mean height is 66 inches ($\mu \neq 66$)

NOV 2003

(9) The heights of 10 males of a given Locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 66 inches. is it reasonable to believe that the average height is greater than 64 inches? (Table value 1.83)

Sol: Given heights of 10 males is

70, 67, 62, 68, 61, 68, 70, 64, 66,

$n = 10$

$\bar{x} = \frac{\Sigma x}{n} = \frac{596}{10}$

$\bar{x} = 59.6$

$\boxed{S^2 = \frac{\Sigma x^2 - n(\bar{x})^2}{n-1}}$

$S^2 = \frac{39554 - 10(59.6)^2}{10-1}$

$S^2 = \frac{39554 - 10(3552.16)}{9}$

$S^2 = \frac{39554 - 35521.6}{9}$

$S^2 = \frac{4032.4}{9} = 448.044$

$S = 21.167$

x	x^2
70	4900
67	4489
62	3844
68	4624
61	3721
68	4624
70	4900
64	4096
66	4356
$\Sigma x = 596$	$\Sigma x^2 = 39554$

$\boxed{\mu = 64}$ Given

2021/3/31 12:25

I.

$H_0: \mu = 64$

$H_1: \mu > 64$ (RT 't' test)

II. <u>C.T.S</u>

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \qquad s/\sqrt{n} = \frac{21.167}{\sqrt{10}} = 6.6935$$

$$t = \frac{59.6 - 64}{6.6935} = \frac{-4.4}{6.6935} = -0.657$$

III. <u>L.O.S</u>

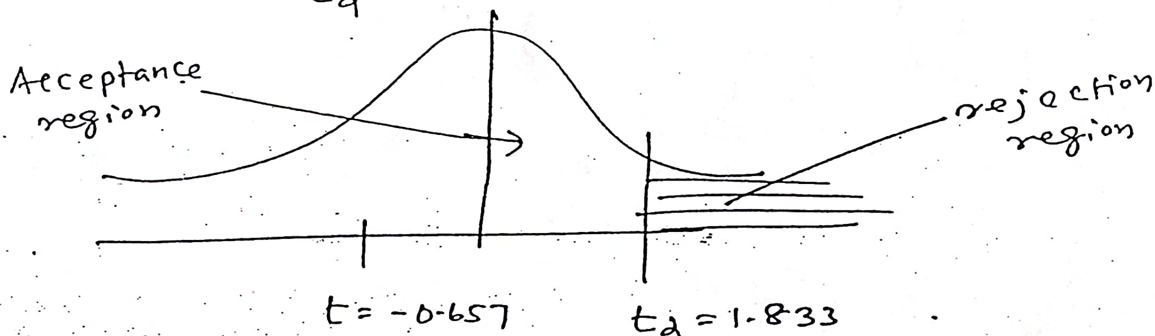
$\alpha = 10\%$

IV. <u>C.V</u>

$df = n-1 = 10-1 = 9$

C.V for RT 't' test at 9 df at $\alpha = 0.10$

$t_\alpha = 1.833$



V. <u>Decision:</u>

Since 't' lies in acceptance region

Ho accepted with $\alpha = 10\%$.

mean height of The males is 64 inches

9) Prices of shares (in Rs) of a Company on different days in a month are given as 66, 65, 64, 63, 61, 60, 59, 71, 72, 67. Will you conclude that the mean price of the share is Less than Rs 65 in a month.

Hint: $n = 10$

$\bar{x} = 64.8$

$s^2 = 19.07$

$s = 4.367$

take $\alpha = 0.05$

$df = 10 - 1 = 9$

$$\boxed{s^2 = \frac{\Sigma x^2 - n(\bar{x})^2}{n-1}}$$

$H_0: \mu = 65$

$H_1: \mu < 65$ (LT 't' test)

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n}}}$$

$t = -0.1447$

(Accept)

10) APRIL 2003

A group of 10 students have an average height of 158 cms. Assuming the heights to follow normal distribution with variance 9 cms test the hypothesis that the mean height of students is 155 cms.

Hint: $n = 10$

~~$\bar{x} = 158$ cms~~

$\sigma^2 = 9$

$H_0: \mu = 155$

$H_1: \mu \neq 155$ (TT 't' test)